Relación 1 · Geometría Diferencial

· Ejercicio pág 23:

$\alpha(t) = x(t), y(t) \qquad t\,[0, 2\pi] \qquad x(t) = a\cos^3 t \qquad y(t) = a\,\text{sen}^3 t$

$x'(t) = 3a\cos^2 t\cdot(-\text{sen}\,t)$

$\alpha'(t) = 3a\cos^2 t\cdot(-\text{sen}\,t),\ 3a\,\text{sen}^2 t\cdot(\cos t) \qquad y'(t) = 3a\,\text{sen}^2 t\cdot(\cos t)$

$$L = 4\cdot\int_0^{\pi/2}\sqrt{[3a\cos^2 t(-\text{sen}\,t)]^2 + [(3a\,\text{sen}^2 t\,(\cos t))]^2}\,dt = \int\sqrt{9a^2\cos^4 t\cdot\text{sen}^2 t + 9a^2\,\text{sen}^4 t\cdot\cos^2 t\,dt}$$

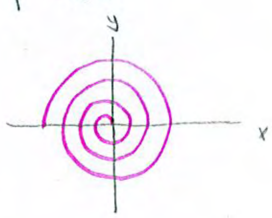
$$\int_0^{\pi/2}\sqrt{9a^2(\cos^4 t\cdot\overbrace{\text{sen}^2 t}) + (\text{sen}^4 t\cdot\overbrace{\cos^2 t})} = \int_0^{\pi/2}\sqrt{9a^2\cdot(\cos^2 t\cdot\text{sen}^2 t)\overbrace{(\cos^2 t + \text{sen}^2 t)}^{1}} =$$

$$\int_0^{\pi/2}\sqrt{9a^2\cdot\cos^2 t\cdot\text{sen}^2 t} = \int_0^{\pi/2} 3a\cos t\cdot\text{sen}\,t\,dt = 4\cdot 3a\int_0^{\pi/2}\text{sen}\,t\cos t\,dt = \left[\frac{\text{sen}^2 t}{2}\right]_0^{\pi/2}$$

$$= 12a\left[\frac{\text{sen}^2\,\pi/2}{2}\right] = 12a\cdot\frac{1}{2} = \boxed{6a}$$

· Ejercicio pág 25:

$\rho = a\theta \qquad \theta\in[0, 2n\pi] \qquad \rho' = a$



$$L = \int_0^{2n\pi}\sqrt{a^2\theta^2 + a^2}\,d\theta = \int_0^{2n\pi}\sqrt{a^2}$$

· Ejercicio pág 28:

Reparametriza: $\alpha[\varphi(s)]$

$\alpha(t) = (\sqrt{t}, t\sqrt{t}, 1-t) \rightarrow (s, s \cdot s^2, 1-s) \rightarrow \boxed{(s, s^3, 1-s)}$

$\sqrt{t} = s \qquad s^2 = t$

· Ejercicio pág 31:

Reparametriza respecto a longitud de arco:

$\alpha(t) = (r\cos t, r\,\text{sen}\,t)$

$\alpha'(t) = (-r\,\text{sen}\,t, r\cos t)$

$s(t) = \int_0^t \| \alpha'(t) \| dt =$

$s(t) = \int_0^t \sqrt{r^2 \underbrace{\text{sen}^2 t + \cos^2 t}_{1}} = \int_0^t \sqrt{r^2}\,dt = r \cdot t$

$s = r\,t \qquad t = \frac{s}{r}$

$\boxed{\beta(s) = r\cos\left(\frac{s}{r}\right) + r\,\text{sen}\left(\frac{s}{r}\right)}$

· Ejercicio pag 35.

Parametrizar la helice por l. de arco:

$\alpha(t) = (a\cos t, a\,\text{sen}\,t, bt) \qquad \alpha'(t) = (-a\,\text{sen}\,t, a\cos t, b)$

$s(t) = \int_0^t \sqrt{a^2\text{sen}^2 t + a^2\cos^2 t + b^2} = \int_0^t \sqrt{a^2 \underbrace{(\text{sen}^2 t + \cos^2 t)}_{1} + b^2}\,dt = \int_0^t \sqrt{a^2+b^2}\,dt =$

$* \left[c = \sqrt{a^2+b^2} \right] \qquad \int_0^t c\,dt = c \cdot t = s \qquad t = \frac{s}{c}$

$\boxed{\beta(s) = \left(a\cos\left(\frac{s}{c}\right) + a\,\text{sen}\left(\frac{s}{c}\right) + b\left(\frac{s}{c}\right)\right)}$

$\vec{X}_\varphi \times \vec{X}_\theta = (-a\,\text{sen}\,\varphi\cos\theta, -b\,\text{sen}\,\varphi\,\text{sen}\,\theta, c\cos\varphi) \times (-a\cos\varphi\,\text{sen}\,\theta, b\cos\varphi\cos\theta, 0) =$

$-a\,(\text{sen}\,\varphi\cos\varphi\,\text{sen}\,\theta\cos\theta) + \cos\theta(-ab\,\text{sen}\,\varphi\cos\varphi) + \text{sen}\,\theta(ba\,\text{sen}\,\varphi\cos\varphi)$

$+\cos\varphi(-ac\,\text{sen}\,\theta) + \cos\varphi(bc\cos\theta)$

$\cos\varphi(ac\,\text{sen}\,\theta + bc\cos\theta)$

1. $\alpha(t) = (r\cos t, r\,\text{sen}\,t)$ $\alpha'(t) = (-r\,\text{sen}\,t, r\cos t)$

$$s(t) = \int_0^t \|\alpha'(t)\| = \int_0^t \sqrt{\underbrace{r^2\text{sen}^2 t + r^2\cos^2 t}_{r^2(\cancel{\cos^2 t + \text{sen}^2 t})\,1}}\,dt = \int_0^t \sqrt{r^2}\,dt = rt \Rightarrow t = \frac{s}{r}$$

$$\boxed{\beta(s) = r\cos\left(\frac{s}{r}\right), r\,\text{sen}\left(\frac{s}{r}\right)}$$

2. $\alpha(s) = \left(\cos\frac{s}{\sqrt{2}}, \text{sen}\,\frac{s}{\sqrt{2}}, \frac{s}{\sqrt{2}}\right)$ si está parametrizada por L. de arco $\|\alpha'(s)\| = 1$

$$\boxed{\alpha'(s) = \left(-\frac{1}{\sqrt{2}}\,\text{sen}\,\frac{s}{\sqrt{2}}, \frac{1}{\sqrt{2}}\cos\frac{s}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right) = \vec{t}}$$

$$\|\alpha'(s)\| = \sqrt{\frac{1}{2}\,\text{sen}^2\frac{s}{\sqrt{2}} + \frac{1}{2}\cos^2\frac{s}{\sqrt{2}} + \frac{1}{2}} = \sqrt{\frac{1}{2}\left(\cancel{\text{sen}^2\frac{s}{\sqrt{2}} + \cos^2\frac{s}{\sqrt{2}}}^{\,1} + 1\right)} = \sqrt{\frac{1}{2}(1+1)}$$

$$\sqrt{\frac{2}{2}} = \sqrt{1} = \boxed{1}$$ si está parametrizada por L. de arco.

$$\alpha''(s) = \left(-\frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}}\cos\frac{s}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\cdot\frac{1}{\sqrt{2}}\,\text{sen}\,\frac{s}{\sqrt{2}}, 0\right)$$

$$\|\alpha''(s)\| = \sqrt{\frac{1}{4}\cos^2\frac{s}{\sqrt{2}} + \frac{1}{4}\,\text{sen}^2\frac{s}{\sqrt{2}}} = \sqrt{\frac{1}{4}\left(\cancel{\text{sen}^2\frac{s}{\sqrt{2}} + \cos^2\frac{s}{\sqrt{2}}}^{\,1}\right)} = \sqrt{\frac{1}{4}} = \frac{1}{2}$$

Triedro de Frenet

$$\bar{n} = \frac{-1/2\cos\frac{s}{\sqrt{2}} - \frac{1}{2}\,\text{sen}\,\frac{s}{\sqrt{2}}}{1/2} = \boxed{-\cos\frac{s}{\sqrt{2}}\,\bar{i} - \text{sen}\,\frac{s}{\sqrt{2}}\,\bar{j}}$$

$$\bar{b} = (\vec{t}\times\bar{n}) = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ -\frac{1}{\sqrt{2}}\,\text{sen}\,\frac{s}{\sqrt{2}} & \frac{1}{\sqrt{2}}\cos\frac{s}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \cos\frac{s}{\sqrt{2}} & \text{sen}\,\frac{s}{\sqrt{2}} & 0 \end{vmatrix} = -\frac{1}{\sqrt{2}}\,\text{sen}\,\frac{s}{\sqrt{2}}\,\bar{i} - \left(-\frac{1}{\sqrt{2}}\cos\frac{s}{\sqrt{2}}\right)\bar{j}$$

$$+\left(-\frac{1}{\sqrt{2}}\,\text{sen}^2\frac{s}{\sqrt{2}} - \frac{1}{\sqrt{2}}\cos^2\frac{s}{\sqrt{2}}\right)\bar{k} =$$

$$\frac{1}{\sqrt{2}} \operatorname{sen} \frac{s}{\sqrt{2}} \bar{i} - \frac{1}{\sqrt{2}} \cos \frac{s}{\sqrt{2}} \bar{j} + \frac{1}{\sqrt{2}} \left(\cancelto{1}{\operatorname{sen}^2 \frac{s}{\sqrt{2}} + \cos^2 \frac{s}{\sqrt{2}}} \right) \bar{k} =$$

$$\boxed{\bar{b} = \frac{1}{\sqrt{2}} \operatorname{sen} \frac{s}{\sqrt{2}} \bar{i} - \frac{1}{\sqrt{2}} \cos \frac{s}{\sqrt{2}} \bar{j} + \frac{1}{\sqrt{2}} \bar{k}}$$

Curvatura:

$$\underline{K} = \| \bar{r}''(s) \| = \boxed{1/2}$$

torsión:

$$\frac{\partial \bar{b}}{\partial s} = -\tau \cdot n$$

$$\frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}} \cos \frac{s}{\sqrt{2}} + \frac{1}{\sqrt{2}} \frac{1}{\sqrt{2}} \operatorname{sen} + 0 = \frac{1}{2} \cos \frac{s}{\sqrt{2}} + \frac{1}{2} \operatorname{sen} \frac{s}{\sqrt{2}} = +\tau \left(+\cos \frac{s}{\sqrt{2}} + \operatorname{sen} \frac{s}{\sqrt{2}} \right)$$

$$\boxed{\tau = \frac{1}{2}}$$

3. Curva parametrizada respecto a l. de arco $\beta(s) = \frac{4}{5} \cos(s), 1 - \operatorname{sen}(s), -\frac{3}{5} \cos$

curvatura

$$K(s) = \| \beta''(s) \|$$

$$\beta'(s) = \boxed{-\frac{4}{5} \operatorname{sen}(s), -\cos(s) + \frac{3}{5} \operatorname{sen}(s) = \underline{\vec{t}}}$$

$$\beta''(s) = \boxed{-\frac{4}{5} \cos(s), \operatorname{sen}(s), \frac{3}{5} \cos(s) = \underline{\bar{n}}}$$

$$K(s) = \sqrt{\frac{16}{25} \cos^2(s) + \operatorname{sen}^2(s) + \frac{9}{25} \cos^2(s)} = \sqrt{\cancelto{1}{\frac{25}{25} \cos^2(s) + \operatorname{sen}^2(s)}} = \boxed{1}$$

$$\bar{b} = \bar{t} \times \bar{n} = \begin{vmatrix} \overset{+}{\bar{i}} & \overset{-}{\bar{j}} & \overset{+}{\bar{k}} \\ -\frac{4}{5} \operatorname{sen}(s) & -\cos(s) & \frac{3}{5} \operatorname{sen}(s) \\ -\frac{4}{5} \cos(s) & \operatorname{sen}(s) & \frac{3}{5} \cos(s) \end{vmatrix} = \left[-\frac{3}{5} \cos^2(s) - \frac{3}{5} \operatorname{sen}^2(s) \right] \bar{i}$$

$$-\frac{3}{5} \left(\cancelto{1}{\cos^2(s) + \operatorname{sen}^2(s)} \right)$$

$$- \left[\left(-\frac{4}{5} \operatorname{sen}(s) \frac{3}{5} \cos(s) \right) - \left(-\frac{3}{5} \operatorname{sen}(s) + \frac{4}{5} \cos(s) \right) \right] \bar{j} + \left[-\frac{4}{5} \operatorname{sen}^2(s) - \left(\frac{4}{5} \cos^2(s) \right) \right]$$

$$-\frac{1}{5} \operatorname{sen}^2(s) + \frac{1}{5} \cos^2(s)$$

$$\cancel{+\frac{1}{5} \left(\operatorname{sen}^2(s) + \cos^2(s) \right)}$$

$$-\frac{4}{5} \left(\cancelto{1}{\operatorname{sen}^2(s) + \cos^2(s)} \right)$$

$$\boxed{\bar{b} = -\frac{3}{5}\bar{i} - {}^{4}/_{5}k}$$

$$\frac{d\bar{b}}{ds} = -\tau \cdot \bar{n} \qquad 0 = -\tau \cdot -{}^{4}/_{5} \cdot \cos(s) \ldots$$

Para esto sea 0

τ debe ser 0.

$\boxed{\tau = 0}$ Significa que es una curva plana.

$$R = \frac{1}{K} = \frac{1}{1} \qquad R = 1$$

La única curva posible que sea plana y tenga un radio = 1, constante, es la circunferencia.

4. $x = a\,sen^2(t) \quad y = a\,sen(t)\cos(t) \quad z = a\cos t$

$$\alpha(a\,sen^2(t),\ a\,sen(t)\cos(t),\ z = a\cos(t))$$

$$x^2 + y^2 + z^2 = a^2$$

$$a^2 sen^4(t) + a^2 sen^2(t)\cos^2(t) + a^2\cos^2(t) = a^2$$

$$a^2\left[sen^4(t) + sen^2(t)\cos^2(t) + \cos^2(t)\right]$$

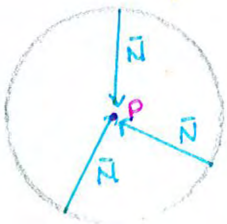
$$\cos^2(t)\left[sen^2 t + 1\right]$$

$$a^2\left[sen^4 t + sen^2(t) + 2\cos^2 t + \cos^2 t\, sen^2 t\right]$$

$$(sen^2 t + \cos t)^2$$

$$sen^4 t + \cos^2 t + 2 sen^2 t \cdot \cos t$$

5. β curva regular $\quad K \neq 0 \quad$ Es una circunferencia



7.

Recta tangente en paramétricas:

$$\left.\begin{array}{l} x = x(t_0) + \lambda x'(t_0) \\ y = y(t_0) + \lambda y'(t_0) \\ z = z(x_0) + \lambda z'(t_0) \end{array}\right\}$$

6. $\alpha(t) = (e^t, e^{-t}, t\sqrt{2})$ $\quad$ Parametrizar respecto a L. de arco $t=0$.

$\alpha'(t) = (e^t, -e^{-t}, \sqrt{2})$ $\qquad (e^{2t}+1)^2$

$$s(t) = \int_0^t \sqrt{e^{2t}+e^{-2t}+2}\, dt = \int_0^t \sqrt{e^{2t}+\frac{1}{e^{2t}}+2}\, dt = \int_0^t \sqrt{\frac{e^{4t}+1+2e^{2t}}{e^{2t}}}\, dt =$$

$$= \int_0^t \sqrt{\frac{(e^{2t}+1)^2}{e^{2t}}}\, dt = \int_0^t \frac{e^{2t}+1}{e^t} = \int_0^t e^t\, dt + \int_0^t \frac{1}{e^t} dt = [e^t]_0^t + [-e^{-t}]_0^t =$$

$e^{-t} dt$

$$(e^t \underbrace{- \cancel{e^0}}_{-1}) + (-e^{-t} \underbrace{- \cancel{(-e^0)}}_{+1}) = e^t - e^{-t} \qquad s(t) = e^t - e^{-t} \quad -e^{-t} = -e^{-t}(-1) = e^{-t}$$

$u = e^t$

$s(t) = e^t - \frac{1}{e^t}$

$$s(t) = u - \frac{1}{u} = \frac{u^2-1}{u} = su = u^2 - 1$$

$u^2 - su - 1 = 0$

$$u = \frac{s \pm \sqrt{s^2 - 4 \cdot 1 \cdot (-1)}}{2} =$$

$$e^t = \frac{s \pm \sqrt{s^2+4}}{2}$$

$\ominus$ NO

$\oplus$

$$e^t = \frac{s+\sqrt{s^2+4}}{2} \qquad \ln e^t = \ln\left(\frac{s+\sqrt{s^2+4}}{2}\right)$$

$$\boxed{t = \ln\left(\frac{s+\sqrt{s^2+4}}{2}\right)}$$

8. Rectas normales — curva $xy=4$ $P(5'5)$

9.

10. Calcula la longitud:

a) $\rho = 3\operatorname{sen}\theta$ $\qquad \theta\ [-\pi/2, \pi/2]$ $\qquad \rho' = 3\cos\theta$

$$S = \int_{-\pi/2}^{\pi/2} \sqrt{9\operatorname{sen}^2\theta + 9\cos^2\theta}\, d\theta = \int_{-\pi/2}^{\pi/2} 3\,(\cancel{\operatorname{sen}^2\theta + \cos^2\theta})\, d\theta = 3\left[\theta\right]_{-\pi/2}^{\pi/2} = 3\left[\pi/2 - (-\pi/2)\right] =$$

$$3\left[\cancel{2}\pi/\cancel{2}\right] = \boxed{3\pi}$$

b) $y = x^2$ $\qquad \begin{cases} x = 0 \\ x = a \end{cases}$

c) $\rho = e^{-\theta}$ $\qquad 0 \leq \theta \leq 2n\pi$ $\qquad \rho' = -e^{-\theta}$

$$S = \int_0^{2n\pi} \sqrt{e^{-2\theta} + e^{-2\theta}}\, d\theta = \int_0^{2n\pi} \sqrt{2e^{-2\theta}}\, d\theta = \int_0^{2n\pi} \sqrt{2}\cdot\sqrt{e^{-2\theta}}\, d\theta = \sqrt{2}\int_0^{2n\pi} e^{-\theta}\, d\theta =$$

$$\sqrt{2}\left[-e^{-\theta}\right]_0^{2\pi n} = \sqrt{2}\cdot\left[-e^{-2\pi n} - (-e^0)\right] = \left[-e^{-2\pi n} + 1\right] = \boxed{\sqrt{2}\left[1 - e^{-2n\pi}\right]}$$

d) Longitud de arco:

$$S=\int_{x_0}^{x_1}\sqrt{1+y'(x)^2+z'(x)^2}\,dx =$$

$P(0,0,0)$

$y(3,9,18)$

$\left.\begin{array}{l} y=x^2 \\ z=\frac{2}{3}xy \end{array}\right\}$ $\begin{array}{l} y=x^2 \\ z=\frac{2}{3}x^3 \end{array}$

$y'=2x \quad z'=2x^2$

$$=\int_0^3\sqrt{1+4x^2+4x^4}\,dx = \int_0^3\sqrt{(2x^2+1)^2}\,dx = \int_0^3 2x^2+1\,dx = 2\left[\frac{x^3}{3}\right]_0^3+\left[x\right]_0^3 =$$

$(2x^2+1)^2$

$4x^4+1+4x^2$

$$=2\left[\frac{3^{3}}{3}-\frac{0^3}{3}\right]+\left[3-0\right] = 18+3=\boxed{21}$$

11. Curvatura:

a) $\rho=3\,\text{sen}\,\theta \qquad \theta\in\left[-\pi/2,\pi/2\right]$

$$K=\frac{\rho^2+2\rho'^2-\rho\rho''}{(\rho^2+\rho'^2)^{3/2}}$$

$\rho'=3\cos\theta \qquad \rho'^2=9\cos^2\theta$

$\rho''=-3\,\text{sen}\,\theta \qquad \rho^2=9\,\text{sen}^2\theta$

$$K=\frac{9\,\text{sen}^2\theta+\underbrace{2(9\cos^2\theta)}_{+18\cos^2\theta}-\underbrace{\left[(3\,\text{sen}\,\theta)\cdot(-3\,\text{sen}\,\theta)\right]}_{+9\,\text{sen}^2\theta}}{(9\,\text{sen}^2\theta+9\cos^2\theta)^{3/2}}=\frac{18\,\text{sen}^2\theta+18\cos^2\theta}{9^{3/2}}=\frac{18}{9^{3/2}}$$

$9(\cancel{\text{sen}^2\theta+\cos^2\theta})^{1}$

$9^{1/2}\cdot 9^1 \qquad 1/2+1=3/2$

$$\frac{18}{9\sqrt{9}}=\frac{2\cdot 9}{9\sqrt{9}}=\boxed{\frac{2}{3}}$$

b) $y=x^2$

$y'=2x$

$y''=2$

$$K=\frac{f''(x_0)}{\left[1+f'(x_0)^2\right]^{3/2}}=\boxed{\frac{2}{\left[1+4x^2\right]^{3/2}}}=$$

c) $\rho = e^{-\theta}$ $\quad 0 \leq \theta \leq 2\pi n$ $\quad \rho^2 = e^{-2\theta}$ $\quad \rho' = -e^{-\theta}$

$\rho'^2 = e^{-2\theta}$ $\quad \rho'' = +e^{-\theta}$

$$K = \frac{\rho^2 + 2\rho'^2 - \rho\rho''}{(\rho^2 + \rho'^2)^{3/2}} =$$

$-e^{-2\theta}$

$$K = \frac{\cancel{e^{-2\theta}} + 2e^{-2\theta} - (e^{-\theta} \cdot e^{-\theta})}{(e^{-2\theta} + e^{-2\theta})^{3/2}} = \frac{2e^{-2\theta}}{(2e^{-2\theta})^{3/2}} = \frac{\cancel{2e^{-2\theta}}}{\cancel{2e^{-2\theta}} \sqrt{2e^{-2\theta}}} = \frac{1}{\sqrt{2e^{-2\theta}}} =$$

$$\frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{e^{-2\theta}}} = \frac{1}{\sqrt{2}} \cdot \frac{1}{e^{-\theta}} = \frac{1}{\sqrt{2}} e^{\theta} = \boxed{\frac{e^{\theta}}{\sqrt{2}}}$$

d) $r(t) \begin{cases} x = t \\ y = t^2 \\ z = 2/3 t^3 \end{cases}$ $\quad K(t) = \dfrac{\| r'(t) \times r''(t) \|}{|r'(t)|^3}$

$r'(t) = (1, 2t, 2t^2)$

$r''(t) = (0, 2, 4t)$

$|r'(t)| = \sqrt{1^2 + 4t^2 + 4t^4}$

$(2t^2+1)^2$

Curvatura

$K(t) =$

$$r'(t) \times r''(t) = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 2t & 2t^2 \\ 0 & 2 & 4t \end{vmatrix} =$$

$r'(t)$ $\quad 4t^2 + 4t + 1$

$(2t+1)$

$$= (8t^2 - 4t^2)\vec{i} - (4t)\vec{j} + (2) = 4t^2 - 4t + 2 = \sqrt{16t^4 + 16t^2 + 4}$$

$2t+2$ $\quad 4t^2$

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$4t^2+2$

$$= \sqrt{(4t^2+2)^2} = 4t^2 + 2$$

$$K(t) = \frac{4t^2+2}{(2t^2+1)^3} = \frac{2(2t^2+1)}{(2t^2+1)^{3/2}} = \frac{2}{(2t^2+1)^2} = \boxed{\frac{2}{(2x^2+1)^2}}$$

$\vec{r}'''(t) = (0,0,4)$

· Torsión:

$$\tau(t) = \frac{\| \vec{r}'(t) \cdot [\vec{r}''(t) \times \vec{r}'''(t)] \|}{|\vec{r}'(t) \times \vec{r}''(t)|^2} = \boxed{\frac{8}{(4t^2+2)^2}} =$$

$$\vec{r}''(t) \times \vec{r}'''(t) = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 2 & 4t \\ 0 & 0 & 4 \end{vmatrix} = 8\vec{i} \; (8,0,0)$$

12. Elipsoide $x^2+2y^2+z^2 = 2$ $\quad z = \sqrt{2-x^2-2y^2}$ $\quad P\left(\overset{x}{-\frac{\sqrt{2}}{2}}, \overset{y}{\frac{1}{2}}, \overset{z}{1}\right)$

$$\begin{cases} x = u \\ y = v \\ z = \sqrt{2-u^2-2v^2} \end{cases}$$

$r_{uv} = (u, v, \sqrt{2-u^2-2v^2})$

Base

$$\underline{r_u'} = \left(1, 0, \frac{-2u}{2\sqrt{2-u^2-2v^2}}\right) = \boxed{\left(1, 0, \frac{-u}{\sqrt{2-u^2-2v^2}}\right)}$$

Base

$$\underline{r_v'} = \left(0, 1, \frac{-4v}{2\sqrt{2-u^2-2v^2}}\right) = \boxed{\left(0, 1, \frac{-2v}{\sqrt{2-u^2-2v^2}}\right)}$$

$$r_v' = \left(0, 1, \frac{-1}{\sqrt{2-\left(-\frac{\sqrt{2}}{2}\right)^2 - 2\left(\frac{1}{2}\right)^2}}\right) = \left(0, 1, \sqrt{2-\frac{2}{4}-\frac{2}{4}}\right) =$$

$-\frac{1}{2} \quad -\frac{1}{2}$

Base en el pto.

$$\boxed{r_v' = (0,1,-1)}$$

Base en el punto

$$\left(1, 0, \frac{+\sqrt{2}/2}{\sqrt{2-|}}\right) = \boxed{\left(1, 0, \sqrt{2}/2\right)} = r_u'$$

Plano tangente

$$\begin{vmatrix} x-x_0 & y-y_0 & z-z_0 \\ (x_u')_0 & (y_u')_0 & (z_u')_0 \\ (x_v')_0 & (y_v')_0 & (z_v')_0 \end{vmatrix} \begin{matrix} \leftarrow \text{Punto} \\ \\ \leftarrow \text{Base en el punto} \end{matrix} \quad \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ x-\frac{\sqrt{2}}{2} & y-\frac{1}{2} & z-1 \\ 1 & 0 & \sqrt{2}/2 \\ 0 & 1 & -1 \end{vmatrix} =$$

$$(z-1)-\left[-\left(y-\tfrac{1}{2}\right)+\left(x+\tfrac{\sqrt{2}}{2}\cdot\tfrac{\sqrt{2}}{2}\right)\right]=z-1\left[-y+\tfrac{1}{2}+x+\tfrac{2}{4}\right]$$

$$z-1+y-\tfrac{1}{2}+x+\tfrac{1}{2}$$

Vector normal

$$\bar{N}=\frac{(\bar{r}'_u\times\bar{r}'_v)}{|\bar{r}'_u\times\bar{r}'_v|}\qquad \bar{r}'_u\times\bar{r}'_v=\begin{vmatrix}\bar{i}&\bar{j}&\bar{k}\\1&0&\frac{\sqrt{2}}{2}\\0&1&-1\end{vmatrix}=\frac{\sqrt{2}}{2}\bar{i}+1-\frac{\sqrt{2}}{2}\bar{j}$$

$$\frac{\sqrt{2}}{2}\bar{i}+\bar{j}+\bar{k}$$

$$\left(x+\frac{\sqrt{2}}{2}\right)\left(-\frac{\sqrt{2}}{2}\right)-\left(y-\frac{1}{2}\right)\cdot(-1)+(z-1)\cdot(1)$$

$$\sqrt{\frac{2}{4}}\ \sqrt{\frac{1}{2}+1+1}=\sqrt{5/2}$$

$$-\frac{x\sqrt{2}}{2}+\left(-\frac{2}{4}\right)\rightarrow-\frac{x\sqrt{2}}{2}\cdot\frac{1}{2}\qquad \frac{x\sqrt{2}}{4}\bar{i}\quad -y+\frac{1}{2}\bar{j}+(z-1)$$

$$\frac{-x\sqrt{2}-4y+2+4z-4}{4}$$

$$\left(y-\frac{1}{2}\right)(-1)$$

$$(z-1)-\left[-y+\frac{1}{2}+\frac{x\sqrt{2}}{2}+\frac{2}{4}\right]$$

$$(z-1)+y-\frac{1}{2}-\frac{x\sqrt{2}}{2}-\frac{1}{2}$$

$$z-1+y-1-\frac{x\sqrt{2}}{2}\qquad z-2+y-\frac{x\sqrt{2}}{2}$$

Plano tangente

$$\boxed{2z-4+2y-x\sqrt{2}=0}$$

$$\bar{N}=\frac{\frac{\sqrt{2}}{2}\bar{i}+\bar{j}+\bar{k}}{\sqrt{5}/\sqrt{2}}=\frac{\frac{\sqrt{2}}{2}}{\sqrt{5}/\sqrt{2}}\bar{i}+\frac{1}{\sqrt{5}/\sqrt{2}}\bar{j}+\frac{1}{\sqrt{5}/\sqrt{2}}\bar{k}$$

Vector normal

$$\frac{2}{2\sqrt{5}}=\boxed{-\frac{1}{\sqrt{5}}\bar{i},\frac{\sqrt{2}}{\sqrt{5}}\bar{j}+\frac{\sqrt{2}}{\sqrt{5}}}$$

15. $\alpha(t) = (t, 1, t^2 - t)$ $\beta(t) = (t-1, 1-t, 2t^2 - 4t + 2)$

contenidas en $z = x(x-y)$

a) $\alpha \begin{cases} x = t \\ y = 1 \\ z = t^2 - t \end{cases}$ $y = 1 - t$ $1 = 1 - t$

$t = 1 - 1 = 0$

$t = 0$

$\beta(P) = \begin{cases} x = 0 - 1 = -1 \\ y = 1 \\ z = 2 \end{cases}$

$t = -1$

$\alpha \begin{cases} x = -1 \\ y = 1 \\ z = (-1)^2 - (-1) = 2 \end{cases}$

$P(-1, 1, 2)$

$z = x(x-y)$

$t^2 - t = t(t-1) = t^2 - t$

$z = x(x-y)$

$2t^2 - 4t + 2 = t - 1[(t-1) - (1-t)] =$

$(2t - 2)(t - 1)$

$2t^2 - 2t - 2t + 2$

$2t^2 - 4t + 2$

a) cos del angulo que forman

$t_\alpha = \frac{\vec{r}'(t_0)}{|\vec{r}'(t_0)|}$

$\alpha'(t) = (1, 0, 2t - 1)$

$|\alpha'(t)| = \sqrt{1^2 + (2t - 1)^2}$

$\beta'(t) = (1, -1, 4t - 4)$

$\beta'(t_0) = (1, -1, -4)$

$\beta'(t_0) = \sqrt{1 + 1 + 16} = \sqrt{18} = 3\sqrt{2}$

$\alpha'(t_0) = 1, 0, -2 - 1 \rightarrow (1, 0, -3)$

$|\alpha'(t_0)| = \sqrt{1^2 + 9} = \sqrt{10}$

$t_\alpha = \frac{1 i + \cancel{0} \, 2t - 1}{}$

$t_\alpha = \frac{1 + 0 - 3}{\sqrt{10}} = \frac{1}{\sqrt{10}}, 0, \frac{-3}{\sqrt{10}}$

$t_\beta = \frac{1 - 1 - 4}{3\sqrt{2}} = \frac{1}{3\sqrt{2}} \; \frac{-1}{3\sqrt{2}} \; \frac{-4}{3\sqrt{2}}$

Producto escalar $\left(\frac{1}{3\sqrt{2}} \cdot \frac{1}{\sqrt{10}}\right) + 0 + \left(\frac{-3}{\sqrt{10}}\right) \cdot \left(\frac{-4}{3\sqrt{2}}\right) =$

$\frac{1}{3\sqrt{2}\sqrt{10}} + \frac{-12}{\sqrt{10} \cdot 3\sqrt{2}} = \frac{13}{3\sqrt{2}\sqrt{10}} = \boxed{\frac{13}{3\sqrt{20}}}$

b) $z = x(x-y)$ $\qquad P(-1,1,2)$

$$\left.\begin{array}{l} x = u \\ y = v \\ z = u(u-v) \end{array}\right| \qquad \bar r(u,v) = (u, v, \overset{u^2-uv}{u(u-v)})$$

$$\bar r_u' = (1, 0, 2u-v) \qquad \bar r_v' = (0, 1, -u)$$

Bases en el punto

$$\bar r_u'(P) = (1,0,-2-1) = (1,0,-3) \qquad \bar r_v' = (0,1,1)$$

$$\bar r_\alpha'(t_0) = A\cdot \bar r_u' + B\cdot \bar r_v' \qquad\qquad \bar r_\beta'(t_0) = C\cdot \bar r_u' + D\cdot \bar r_v'$$

$$(1,0,-3) = A(1,0,-3) + B(0,1,1) \qquad\qquad (1,-1,-4) = (1,0,-3)C + D(0,1,1)$$

$$A = 1 \quad B = 0 \qquad\qquad C = 1 \quad D = -1$$

Producto escalar

$$\bar r_\alpha'(t_0)\cdot \bar r_\beta'(t_0) = A\cdot C\underbrace{(\bar r_u')^2}_{E} + AD\underbrace{(\bar r_u'\cdot \bar r_v')}_{F} + BC(\bar r_v'\cdot \bar r_u') + BD\underbrace{(\bar r_v')^2}_{G}$$

$$G = 1^2 + 1^2 = 2 \qquad E = (\bar r_u')^2 = (1^2+0+9) = 10 \qquad F = (\cancel{1\cdot 0}, \cancel{1\cdot 0}, 1\cdot(-3)) = -3$$

$$\bar r_\alpha'(t_0)\cdot \bar r_\beta'(t_0) = A\cdot CE + [AD + BC]\cdot F + BDG =$$

$$1\cdot 1\cdot 10 + [1\cdot(-1) + \cancel{0\cdot 1}]\cdot(-3) + \cancel{0\cdot(-1)\cdot 2}$$

$$10 + 3 = \boxed{13}$$

16. Curvatura de Gauss y curvatura media $z = x(x-y)$ $P(-1,1,2)$

$$K = \left(\frac{eg - f^2}{EG - F^2}\right) \qquad H = \frac{1}{2}\left(\frac{Eg + Ge - 2Ff}{EG - F^2}\right)$$

$$r(u,v)\begin{cases} x = u \\ y = v \\ z = u^2 - uv \end{cases}$$

$r'_u = (1, 0, 2u - v)$ $\quad r'_v = (0, 1, -u)$

$r'_u = (1, 0, 2\cdot(-1) - 1)$ $\quad r'_v = (0, 1, 1)$

$r'_u = (1, 0, -3)$

$E = (\bar{r}'_u)^2 = 10 \qquad F = \bar{r}'_u \cdot \bar{r}'_v = -3 \qquad G = (\bar{r}'_v)^2 = 2$

$r''_{uu} = (0,0,2)$

$r''_{uv} = (0,0,-1)$

$r''_{vv} = (0,0,0)$

$$\bar{N} = \frac{(r'_u \times r'_v)}{\| r'_u \times r'_v \|} = \frac{-3}{\sqrt{11}}\bar{i} - \frac{1}{\sqrt{11}}\bar{j} + \frac{1}{\sqrt{11}}\bar{k}$$

$$r'_u \times r'_v = \begin{vmatrix} \overset{+}{\bar{i}} & \overset{-}{\bar{j}} & \overset{+}{\bar{k}} \\ 1 & 0 & -3 \\ 0 & 1 & 1 \end{vmatrix} = -3\bar{i} - 1\bar{j} + 1\bar{k}$$

$$\sqrt{3^2 + 1^2 + 1^2} = \sqrt{11}$$

$$e = (0,0,2)\cdot\left(\frac{-3}{\sqrt{11}} - \frac{1}{\sqrt{11}} + \frac{1}{\sqrt{11}}\right) = \frac{2}{\sqrt{11}}$$

$f = \left(\frac{-1}{\sqrt{11}}\right) \qquad g = 0$

$$K = \frac{\cancel{\frac{2}{\sqrt{11}}}\,0 - \frac{1}{11}}{20 - 9} = \boxed{\frac{-1}{121}}$$

K

$$H = \frac{\frac{4}{\sqrt{11}} - \frac{6}{\sqrt{11}}}{2\left(10 \cdot 2 - (-3)^2\right)} = \frac{\frac{-2}{\sqrt{11}}}{22} = \frac{-2}{22\sqrt{11}} = \boxed{\frac{-1}{11\sqrt{11}}}$$

H

17. $z = \text{sen}(x+y)$ $K=0$ $x+y=k\pi$ $\bar{N}=\text{cte}$

a) $\left.\begin{array}{l} x=u \\ y=v \\ z=\text{sen}(u+v) \end{array}\right\}$ $\bar{r}_{uv} = (u, v, \text{sen}(u+v))$

$\bar{r}_u' = [1, 0, \cos(u+v)]$ $\bar{r}_v' = [0, 1, \cos(u+v)]$

$E = (\bar{r}_u')^2 = (1^2 + \cos^2(u+v)) = 1+\cos^2(u+v)$, $G = 1 + \cos^2(u+v)$

$F = (\bar{r}_u' \cdot \bar{r}_v') = (0+0+\cos^2(u+v)) = \cos^2(u+v)$

$$\bar{N} = \frac{(\bar{r}_u' \times \bar{r}_v')}{|\bar{r}_u' \times \bar{r}_v'|} = \frac{-\cos(u+v)\bar{i} - \cos(u+v)\bar{j} + \bar{k}}{\sqrt{2\cos^2(u+v)+1}}$$

$$\bar{r}_u' \times \bar{r}_v' = \begin{vmatrix} \overset{+}{\bar{i}} & \overset{-}{\bar{j}} & \overset{+}{\bar{k}} \\ 1 & 0 & \cos(u+v) \\ 0 & 1 & \cos(u+v) \end{vmatrix} = -\cos(u+v)\bar{i} - \cos(u+v)\bar{j} + 1\bar{k}$$

$$|\bar{r}_u' \times \bar{r}_v'| = \sqrt{\cos^2(u+v) + \cos^2(u+v) + 1^2} = \sqrt{2\cos^2(u+v)+1}$$

$\bar{r}_{uu}'' = (0, 0, -\text{sen}(u+v))$

$\bar{r}_{uv}'' = (0, 0, -\text{sen}(u+v))$

$\bar{r}_{vv}'' = (0, 0, -\text{sen}(u+v))$

$$e = \frac{-\text{sen}(u+v)}{\sqrt{2\cos^2(u+v)+1}}$$

$f =$ ''

$g =$ ''

$$K = \frac{eg - f^2}{EG - F^2}, \quad eg - f^2 = 0 \quad \boxed{K=0}$$ Puntos parabólicos.

b) $z = \text{sen}(x+y)$ $x+y = k\pi$

$\left.\begin{array}{l} x=u \\ y=v \\ z=0 \end{array}\right\}$ $z = \text{sen}(k\pi) = 0$ $\text{sen}\,\pi = 0 \cdot k = 0$

$\bar{r}_{uv} = (u, v, 0)$ $\bar{r}_u' = (1, 0, 0)$ $\bar{r}_v' = (0, 1, 0)$

$$\bar{r}_u' \times \bar{r}_v' = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{vmatrix} = 1\bar{k}$$

$$\bar{N} = \frac{(\bar{r}_u' \times \bar{r}_v')}{|\bar{r}_u' \times \bar{r}_v'|} = \boxed{\frac{\bar{k}}{1}} = \text{cte} \qquad d\bar{N} = 0$$